TRINITY COLLEGE

IN NOMINE DOMINI

Year 11 Specialist Units 1,2
Test 6 2021

Calculator Free

Complex Numbers, Mathematical Induction

STUDENT'S NAME ____________________

DATE: Monday 20 September **TIME:** 50 minutes **MARKS:** 51

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, 1 A4 page of notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

If one root of a quadratic equation is -2 + 5i, determine the quadratic equation in the form $y = ax^2 + bx + c$

$y = (x - (-2+5i))(x - (-2-5i))$

$= (x + 2 - 5i)(x + 2 + 5i)$

$= x^2 + 2x + 5ix + 2x + 4 + 10i - 5ix - 10i - 25i^2$

$= x^2 + 4x + 29$

2. (4 marks)

The sum of two number is -1 and the product of those numbers is 1. Determine the two numbers.

$a+b=-1$ $ab=1$

$a+\frac{1}{a}=-1$ $b=\frac{1}{a}$

$a^2+a+1=0$

$a=\frac{-1\pm\sqrt{1-4}}{2}$

$=\frac{-1\pm\sqrt{3}i}{2}$

$b=\frac{-1\pm\sqrt{3}i}{2}$

3. (6 marks)

Given $z=5+2i$

(a) determine z^2 [2]

$21+20i$

(b) determine $(\bar{z})^2$ [2]

$21-20i$

(c) describe the relationship between z^2 and $(\bar{z})^2$ [2]

$(\bar{z})^2$ IS THE CONJUGATE OF z

4. (5 marks)

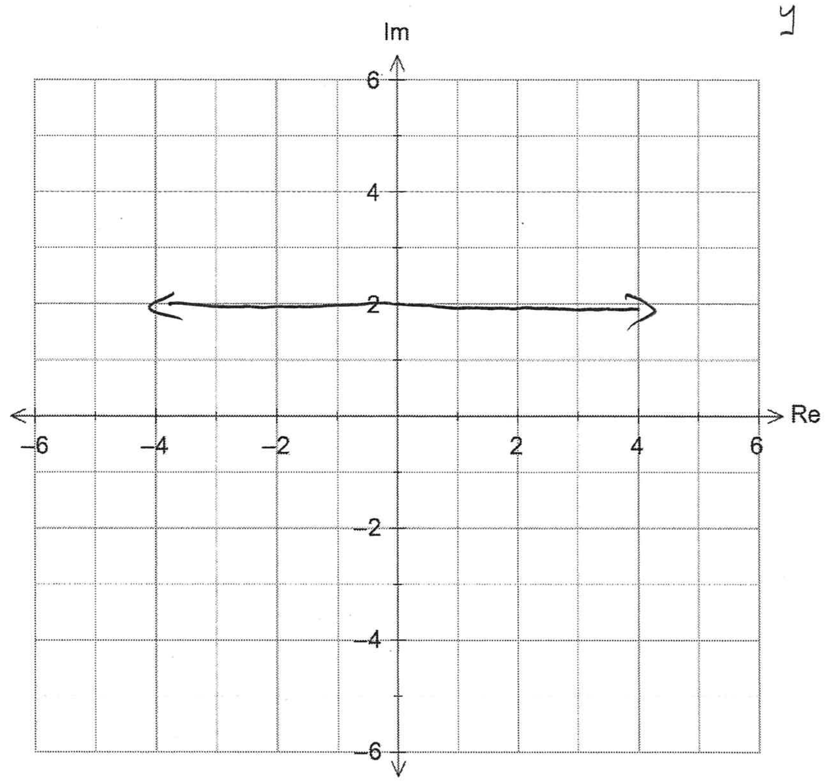
For the complex number z, where $z = x + iy$

$x+iy - (x-iy) = 4i$

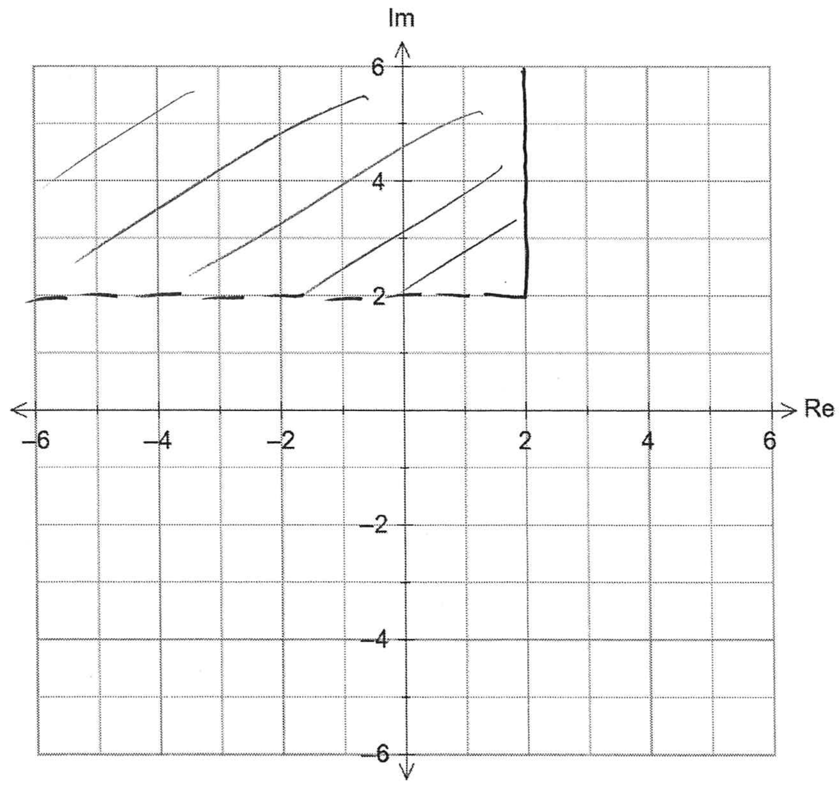
$2iy = 4i$

$y = 2$

(a) Sketch $z - \overline{z} = 4i$ [3]



(b) Sketch $\operatorname{Im} z > 2$ and $\operatorname{Re} z \leq 2$ [2]



5. (8 marks)

Determine the complex number z, in the form $a+bi$, if

(a) $(z-2)^2+3=0$ [4]

$(z-2)^2 = -3$

$z-2 = \pm\sqrt{3}i$

$z = \pm\sqrt{3}i + 2$

(b) $2z+3=i(\bar{z})-5$ [4]

$2x + 2iy + 3 = ix - i^2y - 5$

$2x + 3 + 2iy = y - 5 + ix$

Re $2x + 3 = y - 5$ Im $2y = x$

$4y + 3 = y - 5$ $\frac{-16}{3} = x$

$3y = -8$

$y = -\frac{8}{3}$

$z = \frac{-16}{3} - \frac{8}{3}i$

6. (9 marks)

Given $z = 2 - 5i$ and $w = 1 + 6i$, determine

(a) $iz + \overline{w}$ [3]

$= 2i - 5i^2 + 1 - 6i$

$= 6 - 4i$

(b) $\dfrac{i}{w}$ [3]

$= \dfrac{i}{1+6i} \times \dfrac{1-6i}{1-6i}$

$= \dfrac{6+i}{1+36}$

$= \dfrac{6+i}{37}$

(c) $\text{Im}\left(\dfrac{z}{-i}\right)$ [3]

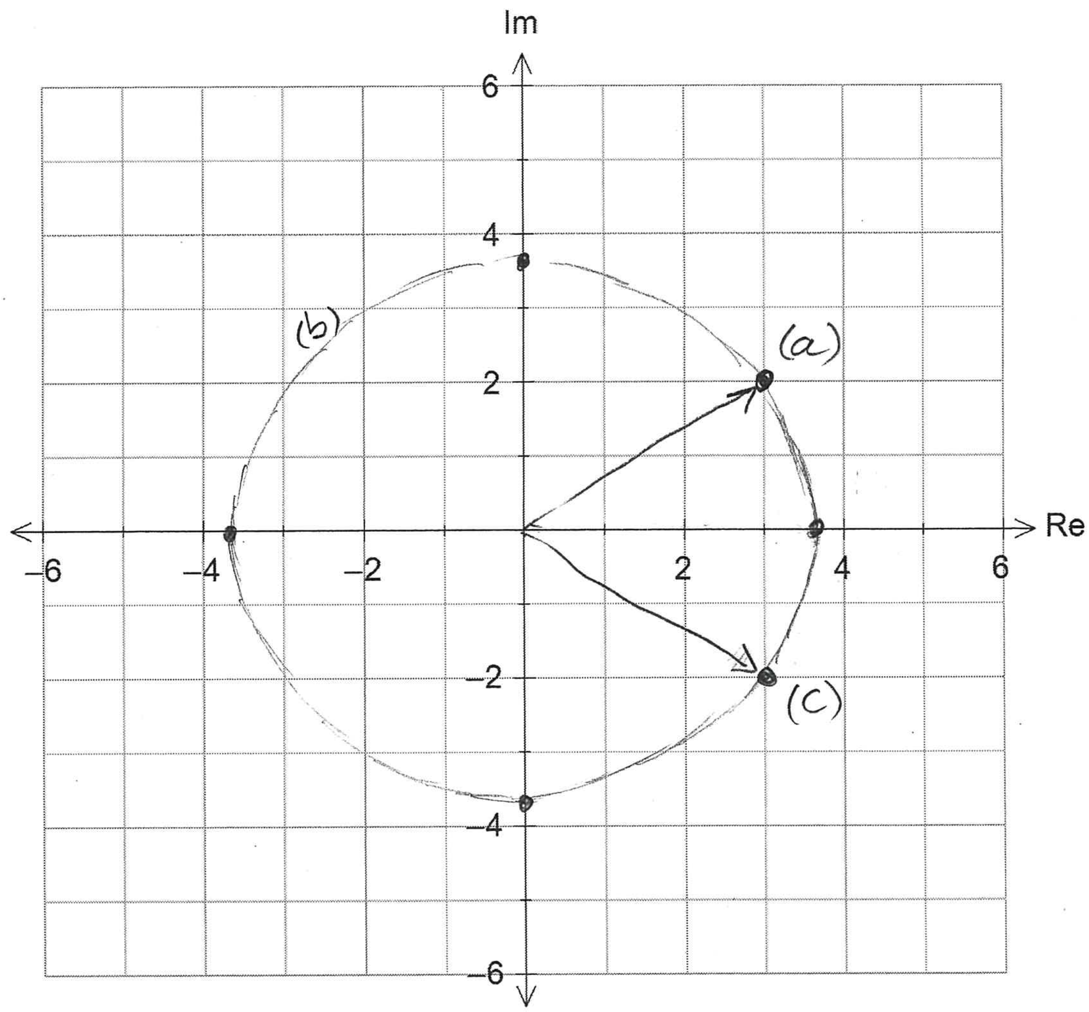
$= \text{Im}\left(\dfrac{2-5i}{-i} \times \dfrac{-i}{-i}\right)$

$= \text{Im}\ (5+2i)$

$= 2$

7. (8 marks)

If $w = -2 + 3i$, on the axes below plot the following.



(a) wi^3 $= -i(-2+3i)$ [3]

$= 3 + 2i$

(b) $|w|$ $= \sqrt{2^2 + 3^2}$ [2]

$= \sqrt{13}$

(c) $\dfrac{\overline{w}}{i^3}$ $= i(-2-3i)$ [3]

$= 3 - 2i$

8. (7 marks)

Use mathematics induction to prove $n! > 2^n$ for n a positive integer greater than or equal to 4.

$n = 4$ $\qquad 4! = 24 \qquad 2^4 = 16$

$4! > 2^4$

ASSUME TRUE FOR $n = k$ $\qquad k \in n$

ie $k! > 2^k$

PROVE TRUE FOR $n = k+1$

ie PROVE $(k+1)! > 2^{k+1}$

$(k+1)!$

$= (k+1)\,k!$

WITH $k! > 2^k$

AND $k+1 > 2$ $\qquad (n \geq 4)$

$(k+1)\,k! > 2 \times 2^k$

$(k+1)! > 2^{k+1}$

SINCE ASSUMED TRUE FOR $n = k$ AND TRUE FOR $n = k+1$

AND TRUE FOR $n = 4$ THEN TRUE FOR $n = 5$

SINCE TRUE FOR $n = 5$ THEN TRUE FOR $n = 6$

AND SO ON

$\therefore$ TRUE FOR $n \geq 4$